Approximating Triangular Meshes by Implicit, Multi-Sided Surfaces

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Outline

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- Previous work

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- Previous work
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- Refinement

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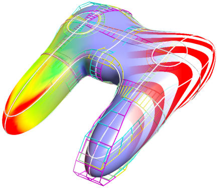
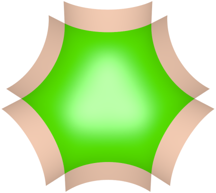
Implicit multi-sided patches

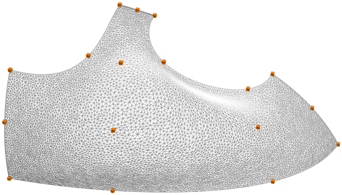
- Multi-sided surfaces used in many areas
  - Design
  - Hole filling & Vertex blending
- Exact representations $\rightarrow$
  - explicit shape control
  - watertight connections
- Generally – n-sided parametric patches
- An interesting alternative: implicit patches
  - Easy inside-outside testing
  - Boolean operations
  - Efficient raytracing
  - Connect to regular implicit surfaces
  - **Suitable for approximation**

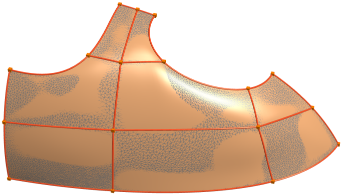




Approximation task

Fitting a smooth mesh with a collection of multi-sided patches in implicit form.



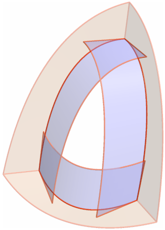


Previous work

- Mostly global implicit approximations:
  - Radial basis functions (Carr'01)
  - Poisson reconstruction (Kazhdan'13)
  - Neural networks (Takikawa'21)
- Very general methods without geometric meaning
- Our approach is closer to implicit modeling schemes [A-patch (Bajaj'95), functional spline (Li'90, Hartmann'01), ...]

I-patch

Basic formula[1]



- $\{R_i(x,y,z)=0\}$ *implicit* ribbons,
  $\{B_i(x,y,z)=0\}$ *implicit* bounding surfaces
- $R_i \cap B_i \Rightarrow i^{\text{th}}$ boundary
- $$I=\sum_{i=1}^{n}\left(w_i R_i \prod_{j\neq i} B_j^{k+1}\right)+w\prod_{i=1}^{n}B_i^{k+1}$$
- Patch: $\{I(x,y,z)=0\}$
- k: degree of continuity
- Free parameters: $w_i, w \in \mathbb{R}$

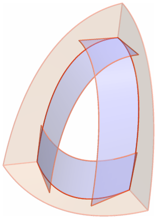
[1] T. Várady, P. Benkő, G. Kós, A. Rockwood: *Implicit surfaces revisited – I-patches.* Geometric Modelling, Springer, Vienna, pp. 323–335, 2001.

Example

3-sided G^1 I-patch:

$$w_1 R_1 B_2^2 B_3^2 + w_2 R_2 B_1^2 B_3^2 + w_3 R_3 B_1^2 B_2^2 + w B_1^2 B_2^2 B_3^2 = 0$$

- On boundary #i, $R_i(x, y, z) = 0$ and $B_i(x, y, z) = 0 \Rightarrow I(x, y, z) = 0$
- On boundary #i, $I = R_i G + B_i^2 H$ $(G, H : \mathbb{R}^3 \rightarrow \mathbb{R}) \Rightarrow \nabla I = const \cdot \nabla P_i$

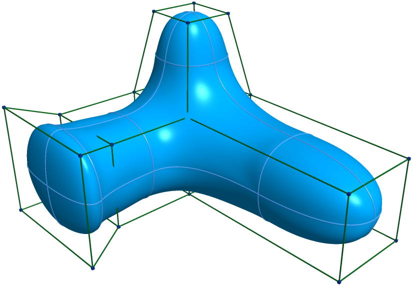


Additional properties

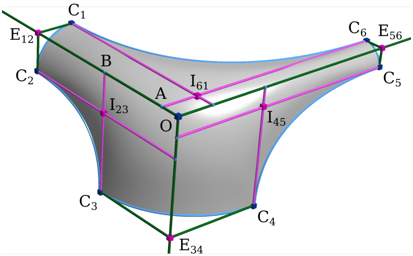
$$I = \sum_{i=1}^{n} \left(w_i R_i \prod_{j \neq i} B_j^{k+1} \right) + w \prod_{i=1}^{n} B_i^{k+1},\ w_i, w \in \mathbb{R}$$

- Interior C^∞
- G^k continuity to all R_i-s along the border
  - B_i-s derivatives do not affect $I^{(m)}, m \leq k$ on the boundary because of $k+1$ exponent
- Coincident bounding surfaces: slightly modified equation

Our previous work[2]



Design with polyhedra



Setback vertex blending

[2] Á. Sipos, T. Várady, P. Salvi, M. Vaitkus: *Multi-sided implicit surfacing with I-patches*. Computers & Graphics 90, pp. 29-42, 2020.

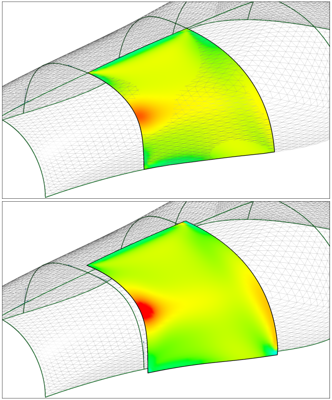
"Faithful" distance fields

- Alternative form of I-patches:

$$I_{orig} = \sum_{i=1}^{n} (R_i \cdot \alpha_i) + \alpha_0$$

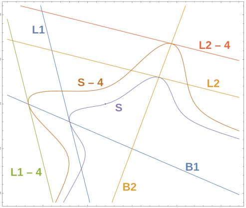
$$I_{faithful} = \frac{\sum_{i=1}^{n} (R_i \cdot \alpha_i) + \alpha_0}{\sum_{i=1}^{n} \alpha_i}$$

- The offset of a faithful I-patch is the same as the I-patch created from the offsets of the ribbons.

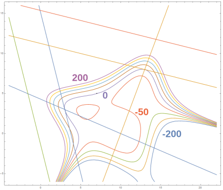


“Faithful” distance fields

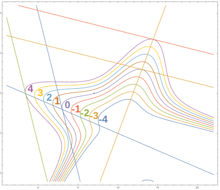
The offset of a faithful I-patch is the same as the I-patch created from the offsets of the ribbons (with the same boundings).



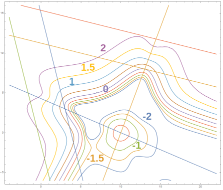
Algebraic distance:



Faithful distance:



Gradient normalization:



"Faithful" distance fields

The offset of a faithful I-patch is the same as the I-patch created from the offsets of the ribbons (with the same boundings).

Proof:

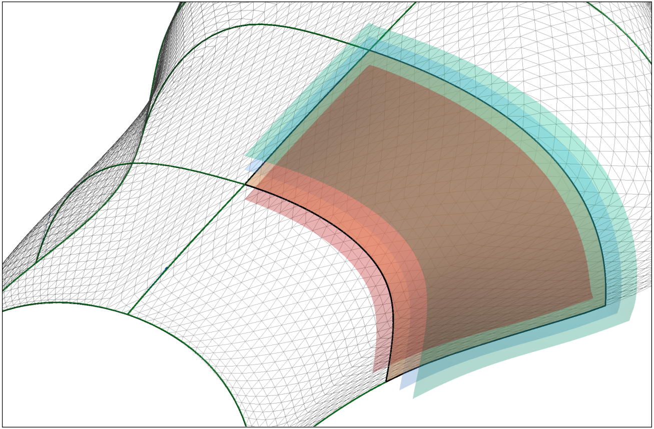
$$I = \frac{\sum_{i=1}^{n}(R_i \cdot \alpha_i) + \alpha_0}{\sum_{i=1}^{n}\alpha_i}$$

$$I - d = \frac{\sum_{i=1}^{n}(R_i \cdot \alpha_i) + \alpha_0}{\sum_{i=1}^{n}\alpha_i} - \frac{d \cdot \sum_{i=1}^{n}\alpha_i}{\sum_{i=1}^{n}\alpha_i}$$

$$I - d = \frac{\sum_{i=1}^{n}((R_i - d) \cdot \alpha_i) + \alpha_0}{\sum_{i=1}^{n}\alpha_i}$$

“Faithful” distance fields

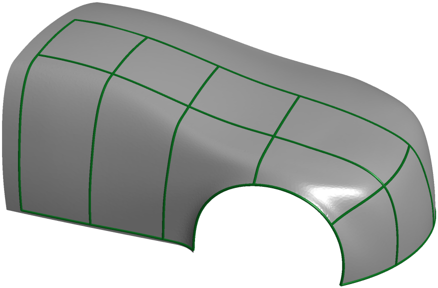
A good distance measure



Approximation with I-patches

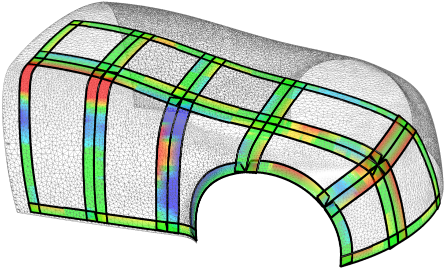
Input

- A (smooth) triangle mesh
- A vertex graph on this mesh



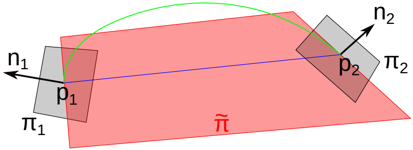
Construction – overview

- In each vertex: normal vector estimation from mesh
- On each edge:
  - A ribbon surface blending the corner planes
  - A bounding surface intersecting the mesh between the corners (broadly) perpendicularly
- Each face of the graph is filled with a patch

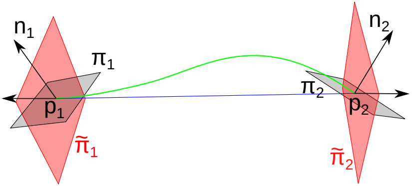


Construction – ribbons

- A blend of the corner planes
- Approximated on sample points along the bounding and the mesh



Liming-ribbon



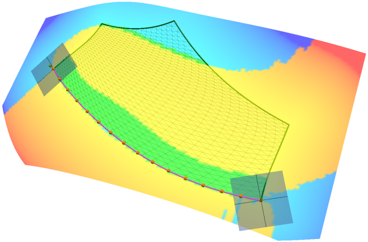
I-loft ribbon

Construction – boundings

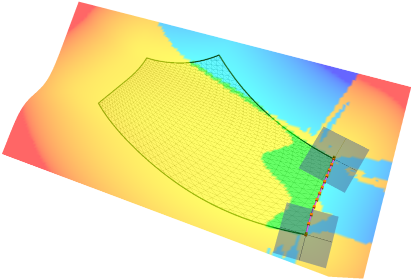
- Internal edge ⇒ a plane
  - containing the corners and the average of the normals
- Boundary edge ⇒ curved bounding
  - same kinds of blend surfaces as ribbons

Construction – ribbon approximation

Ribbons (and curved boundings) are approximating sample points along the bounding and the mesh



Liming-ribbon



I-loft ribbon

"Better" is chosen (criteria: 1. normal consistency, 2. algebraic error)

Setting the optimal weights

$$I = \sum_{i=1}^{n} \left(w_i R_i \prod_{j \neq i} B_j^{k+1} \right) + w \prod_{i=1}^{n} B_i^{k+1}$$

- R_i, B_i are fixed by this point
- Only scalars (w_i) need to be set

Cost function:

$$\arg\min_{\mathbf{w}} \sum_{i=1}^{N} I_{\mathbf{w}}^2(\mathbf{q}_i), \text{ s.t. } \max_{i,j} \left(\frac{w_i R_i(\mathbf{c})}{B_i^2(\mathbf{c})} \Big/ \frac{w_j R_j(\mathbf{c})}{B_j^2(\mathbf{c})} \right) \leq \omega,$$

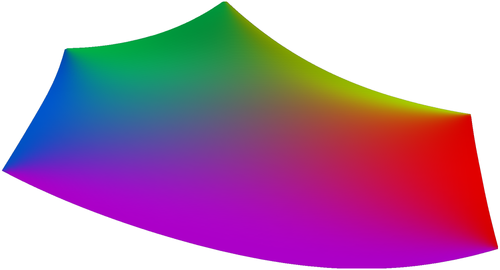
where $\mathbf{q}_i$ are the data points to be approximated, $\mathbf{c}$ is the central point of the patch.

Setting the optimal weights – constraining

Cost function:

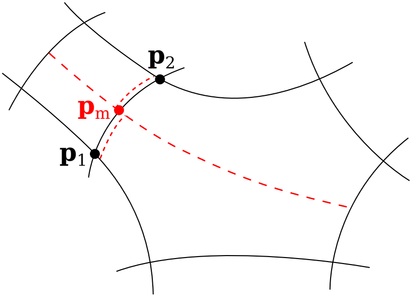
$$\arg\min_{\mathbf{w}} \sum_{i=1}^{N} I_{\mathbf{w}}^2(\mathbf{q}_i), \text{ s.t. } \max_{i,j} \left(\frac{w_i R_i(\mathbf{c})}{B_i^2(\mathbf{c})} \Big/ \frac{w_j R_j(\mathbf{c})}{B_j^2(\mathbf{c})} \right) \leq \omega,$$

The constraint was introduced to prevent one side interpolant dominating others. (ω is a user parameter)

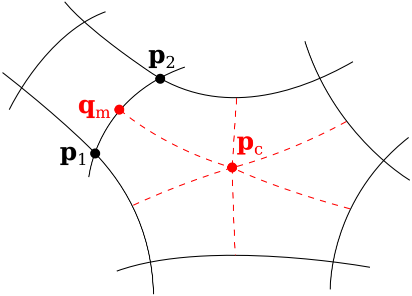


Adaptive refinement

Approximation error too high $\Rightarrow$ subdivide the patches



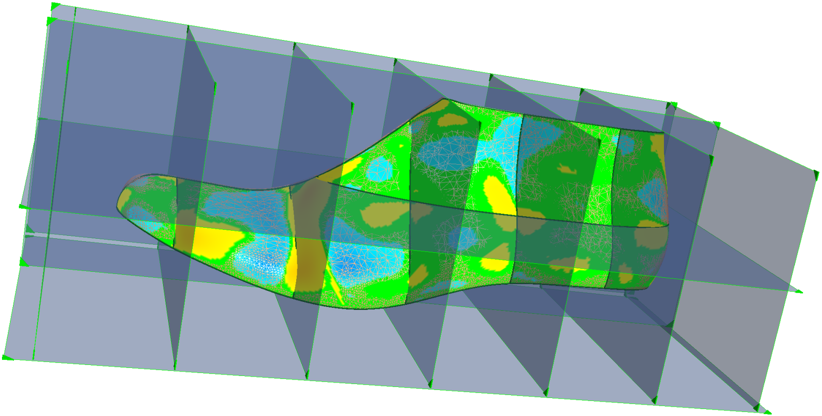
Splitting ribbon and both patches



Splitting a single patch while keeping ribbon (T-node)

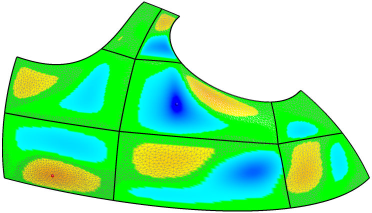
Examples

Shoe last

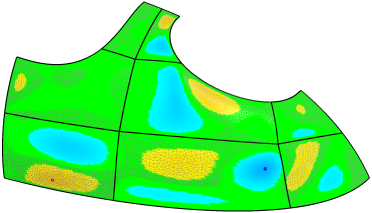


Average deviation: 0.069%, maximum deviation: 0.35%

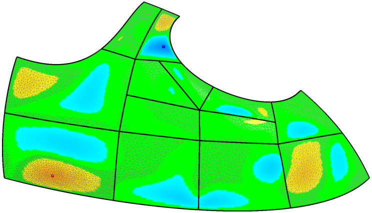
Sheet metal



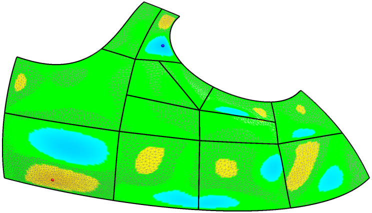
avg.: 0.055%, max.: 0.399%



avg.: 0.035%, max.: 0.226%

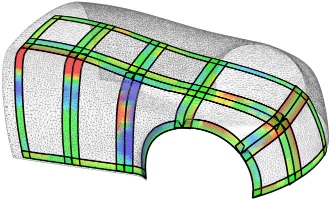


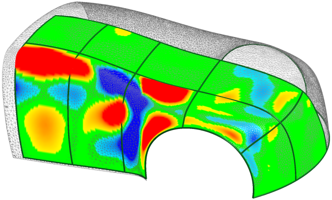
avg.: 0.041%, max.: 0.283%



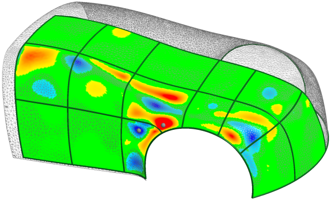
avg.: 0.029%, max.: 0.175%

Concept car

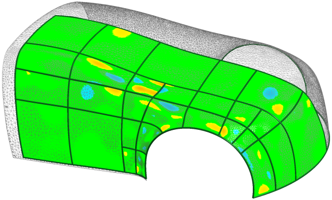




avg.: 0.140%, max.: 3.000%



avg.: 0.027%, max.: 0.300%



avg.: 0.016%, max.: 0.120%

Conclusion

Summary

I-patches

- Suitable for multi-sided modeling
- Faithful distance field

Approximation

- Constructing ribbon/bounding surfaces
- Setting interior parameters
- Refining the patchwork

Questions?

Thank you for your attention.

Contact: *asipos@edu.bme.hu*